



Deteriorated turbulent heat transfer (DTHT) of gas up-flow in a circular tube: Heat transfer correlations

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ABSTRACT

This paper presents comparisons of recent experimental data, presented in a companion paper, to the existing correlations for heat transfer in various gas flow regimes and development of more accurate new correlations. The existing correlation of Celeta et al. showed the best agreement with the data in the turbulent heat transfer regime, while most of the existing correlations for laminar heat transfer showed unsuccessful predictions. Three new sets of correlations, each covering the mixed convection laminar, normal turbulent and deteriorated turbulent heat transfer (DTHT) regimes for heated gas up-flow, have been developed to agree better with the data in each regime.

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1. Introduction

Experimental data presented in [1–3] show that two major physical phenomena: (1) buoyancy and (2) acceleration, may hinder normal turbulent heat transfer in gas flowing upwards in a heated channel. This regime is called deteriorated turbulent heat transfer (DTHT). Furthermore, when a gas flow rate is low enough to fall within the laminar flow regime a high heat flux can enhance laminar heat transfer by generating a strong buoyancy force. These special heat transfer regimes do not follow the convection theory developed from assumptions of constant thermo-physical properties and weak buoyancy force. The predictions based on this theory are often significantly different from the observations.

A review of the existing theories for the buoyancy driven and acceleration driven DTHT will be presented more thoroughly than before [1], and this presentation will be followed by a review of existing correlations relevant to the DTHT regime. Next, the correlations reviewed will be compared to recent experimental data obtained for nitrogen, helium and carbon dioxide gases, which were presented in [1–3]. Finally, development of new correlation sets, which perform better than the correlations reviewed, will be presented.

2. Literature survey

A strong buoyancy force can impact the flow in various ways depending on different combinations of the buoyancy force and

the flow directions. Within this study, only upward heated flow with uniform wall heat flux will be addressed since our recent experimental data [1–3] were obtained for this situation.

The buoyancy effect can alter the heat transfer characteristics of both laminar and turbulent flows. Under laminar flow conditions, the buoyancy force results in a steeper velocity gradient near a strongly heated wall than a weakly heated wall. Therefore, the ability to remove thermal energy from the wall increases and laminar heat transfer is enhanced. Typically, the situation where a strong buoyancy force acts on forced flow is called “Mixed Convection”. A selection of an appropriate non-dimensional number for describing the buoyancy force applied to the laminar flow is important to characterizing the experimental data. When non-dimensionalizing the momentum equation, the temperature in the buoyancy force term, where the Boussinesq approximation is applied, is a controversial variable for selecting the reference parameters, as we will see in a later part of the review. The two-dimensional steady-state momentum equation for constant fluid properties (except in the buoyancy term) is considered for simplicity.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \underbrace{g\beta(T - T_0)}_{\text{Boussinesq approximation}} \quad (1)$$

According to Petukhov and Polyakov [4] (p. 20) the reference parameter for non-dimensionalizing the temperature should be dependent on the boundary conditions. Since only uniform wall heat flux is considered in this paper, a non-dimensional group based on heat flux will be adopted. The resulting non-dimensional momentum equation is presented in Eq. (2).

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Nomenclature

A	area (m ²)	Ra	Rayleigh number ($=Gr_{\Delta T}Pr$)
Bo^*	buoyancy parameter ($=Gr_q/Re^{3.425}Pr^{0.8}$, Ref. [5])	Re	Reynolds number ($=U_b D/\nu$)
c_p	specific heat at constant pressure (J/kg K)	T	temperature (K)
D	pipe diameter (m)	U	velocity (m/s)
E	buoyancy parameter ($=Gr_q/Re^4Pr$, Ref. [4])	x	axial direction and distance (m)
G	mass flux (kg/m ² s)	x^+	non-dimensional axial distance $= 2x/DRePr$
g	gravitational acceleration (m/s ²)	y	radial direction
$Gr_{\Delta T}$	Grashof number ($=g\beta(T_w - T_b)D^3/\nu^2$)	Greek symbols	
Gr_q	Grashof number based on heat flux ($=g\beta q_w''D^4/k\nu^2$)	α	thermal diffusivity ($=k/\rho c_p$, m ² /s)
Gz	Graetz number ($=(\pi D/4x)RePr$)	β	thermal expansion coefficient ($=(-1/\rho)(\partial\rho/\partial T)_p$, K ⁻¹)
H	enthalpy (J/kg)	μ	dynamic viscosity (kg/m s)
h	heat transfer coefficient (W/m ² K)	ν	kinematic viscosity (m ² /s)
K_v	acceleration parameter ($=\nu/U_b^2(dU_b/dx) \approx 4q^+/Re$)	ρ	density (kg/m ³)
k	thermal conductivity (W/m K)	Subscripts	
L	distance from the inlet (m)	b	bulk
L/D	maximum value of x/D	F	forced convection
\dot{m}	mass flow rate (kg/s)	in	inlet
Nu	Nusselt number ($=hD/k$)	N	natural convection
P	system pressure (MPa)	o	reference state
Pr	Prandtl number ($=\nu/\alpha$)	th	threshold
q^+	non-dimensional heat flux	w	wall
q''	heat flux (W/m ²)		
q^*	non-dimensional heat flux ($=Dq_w''/2k_{in}T_{bin}$, Ref. [13])		

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right) + \frac{Gr_q}{Re^2} T^*, \quad (2)$$

where

$$x^* = \frac{x}{D}, \quad y^* = \frac{y}{D}, \quad u^* = \frac{u}{U_b}, \quad v^* = \frac{v}{U_b}, \quad p^* = \frac{p}{\rho_o U_b^2},$$

$$T^* = \frac{(T - T_o)}{T_{ref}}, \quad T_{ref} = \frac{q_w'' D}{k}.$$

For turbulent flow, there are two theories to explain the effects of strong buoyancy on turbulence. One was introduced by Hall and Jackson [5] and the other by Petukhov and Polyakov [4]. Jackson and Hall first developed their theory to explain the heat transfer deterioration observed in turbulent heat transfer in supercritical fluids. Their theory is based on the observation that turbulence is mostly generated and diffused by the shear stress near the boundary between the wall region and the core region. Since the buoyancy force accelerates the flow near the wall relatively more than in the core, the average velocity difference between the wall and the core is reduced. Therefore, the shear stress in the region between the wall and the core is reduced since the shear stress is proportional to the average velocity difference between two regions. As a result, the flow tends to stabilize, causing a decrease in the turbulence production and a consequent decrease in turbulent heat transport. More details can be found in [5]. The resulting governing non-dimensional number that was developed by Jackson and Hall is called the buoyancy parameter Bo^* .

On the other hand, Petukhov and Polyakov [4] adopted concepts for turbulence interaction with buoyancy forces, from the meteorology field [6]. Petukhov and Polyakov suggest that the heating of the upward flow can cause two effects: an external effect and a structural effect. The external effect is defined as the change of the mean velocity field due to the buoyancy force and the structural effect is defined as additional work required for turbulence to overcome the stabilized density gradient in upward heated flow. For high heating rates, the structural effect is much stronger than the external effect, thereby leading to a decrease in the turbulent intensity due to energy loss via work against the stabilized density

distribution. The governing non-dimensional parameter developed within this framework is the buoyancy number E . Both Bo^* and E are defined in Eq. (3).

$$Bo^* = \frac{Gr_q}{Re^{3.425}Pr^{0.8}}, \quad E = \frac{Gr_q}{Re^4 Pr}. \quad (3)$$

In summary, even though these two theoretical frameworks provide different explanations for the mixed convection effect on turbulent flow, the resulting non-dimensional numbers are comparable. Within our study, Bo^* will be used for correlating the experimental data for the buoyancy induced DTHT cases, since it has been applied more widely in the literature for developing correlations than the Petukhov and Polyakov parameter has been utilized.

The second phenomenon which can induce DTHT is the acceleration effect. Flow acceleration has been studied for converging channels, where the mean velocity increases due to reduction in flow area [7]. A strongly heated flow, where the fluid density decrease due to heating causes an increase of the mean flow velocity can exhibit a behavior comparable to converging channels. Thus, strongly heated flow also can be affected by the acceleration effect so that significant reduction of turbulent heat transfer can be observed due to apparent ‘‘laminarization’’ of turbulent flow. To represent this situation an acceleration parameter is defined in Eq. (4), which was derived in Ref. [8]. The transformation of the earlier acceleration parameter to one for heated gas flow was performed by applying energy and mass balance equations with perfect gas and constant cross section assumptions to the original definition

$$K_v = \frac{\nu}{U_b^2} \frac{dU_b}{dz} \approx \frac{4q_b^+}{Re}. \quad (4)$$

In contrast to the buoyancy driven DTHT, there are few heat transfer correlations for the acceleration driven DTHT regime and they are not easily found. This paucity occurs because most literature sources focus on a threshold value for the acceleration driven laminarization process, rather than on development of heat transfer correlations. Therefore, the acceleration induced DTHT papers are not explicitly reviewed in this study. For related information on the estimation of the acceleration DTHT threshold, one may see [9,10].

For the two effects, the threshold values indicating when forced turbulent heat transfer is likely to become DTHT are $Bo_{in}^* \sim 2 \times 10^{-6}$ and $K_{vin} \sim 2.5 \times 10^{-6}$. These values were updated in [1–3] from the recommendations by McEligot and Jackson [9] ($Bo^* \sim 6 \times 10^{-7}$ and $K_v \sim 3 \times 10^{-6}$). It should be emphasized that not only the threshold values were updated for separating the normal forced turbulent regime from the DTHT regime but also the location where the parameters should be evaluated was revised from [9] to [1–3]. Here the threshold value is evaluated at the inlet of the channel rather than at a local point in the channel, since choosing the inlet value seems to make a clearer demarcation between the normal forced turbulent and the DTHT regimes for these data, as was discussed in [1–3]. Even though the onset of each DTHT regime is reasonably well defined, the predicted heat transfer coefficient for this regime differs among the researchers, as will be shown in the following sections. It should be noted that all non-dimensional numbers presented here are evaluated from the local bulk fluid properties, unless it is specified otherwise.

2.1. Laminar convection with large buoyancy effect

2.1.1. Hallman (1961)

Hallman solved for the fully developed laminar velocity and temperature profiles analytically when the forced and free convection forces have comparable magnitude [11]. He used as the governing non-dimensional number the Rayleigh number defined with the axial temperature gradient. If the axial temperature gradient is transformed as a function of heat flux by applying an energy balance, Hallman's definition of the Rayleigh number can be represented with a combination of the Grashof number based on heat flux and Reynolds number. Hallman [12] presented experimental data from water experiments as well as a modified heat transfer correlation, which was based on his earlier theoretical development. The correlation is believed to be valid for the range of $100 \leq Ra_{dT_b/dx} = (Gr_q/4Re) \leq 10,000$

$$Nu_{Hallman} = 1.40 Ra_{dT_b/dx}^{0.28} = 1.40 \left(\frac{Gr_q}{4Re} \right)^{0.28} \quad (5)$$

2.1.2. Worsøe-Schmidt and Leppert (1965)

Worsøe-Schmidt and Leppert conducted a numerical analysis [13]. They developed an implicit finite difference scheme for solving the governing equations for laminar gas flow in a vertical, heated circular tube with large variations of gas properties. Based on their numerical scheme, friction factor and Nusselt number correlations were given for air with both uniform wall heat flux and uniform wall temperature boundary conditions. Later, Worsøe-Schmidt applied the same finite difference scheme for helium and carbon dioxide and proposed heat transfer correlations based on the results [14], as shown in Eq. (6).

When $3 < Gz < 1000$, $0 < q^* \leq 20$,

$$Nu_{Worsøe-schmidt} = 4.36 + \frac{0.025 q^{*1/2} (Gz - 3)(Gz - 20)}{Gz^{3/2}}, \quad (6)$$

when $Gz < 3$, $Nu_{Worsøe-schmidt} = 4.36$.

2.1.3. Churchill (1988)

Churchill presented a correlation for laminar mixed convection by combining a free convection correlation with a forced convection correlation for the uniform heat flux case [15]. The correlation was tested against Hallman's data and some numerical results,

$$Nu_{Churchill}^6 = Nu_F^6 + Nu_N^6, \quad \text{where } Nu_F = \frac{48}{11} \approx 4.364, Nu_N = 0.846 (Ra_{dT_b/dx})^{1/4} = 0.846 \left(\frac{Gr_q}{4Re} \right)^{1/4} \quad (7)$$

2.2. Buoyancy-induced DTHT (mixed convection)

In this section, forced convection Nusselt numbers (Nu_F) will be calculated from the Gnielinski correlation [16] (Eq. (8)), even though the original correlations for mixed convection were developed from different forced convection correlations. Since most general forced convection correlations overlap each other within a few percent, the differences between the original correlation and the Gnielinski-based correlation will be small.

$$Nu_{Gnielinski} = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b} \right)^{-0.45} \left(1 + \left(\frac{L}{D} \right)^{-2/3} \right),$$

where $f = (1.82 \log_{10} Re - 1.64)^{-2}$. (8)

2.2.1. Petukhov and Polyakov (1988)

From their theoretical development, which was summarized previously, a semi-empirical correlation was developed in [4]. The claimed validity of the correlation is for a Reynolds number above 3000, heat flux-based Grashof number below 10^{11} and L/D longer than 40. The correlation was tested against experimental data from other literature sources.

$$Nu_{Petukhov-Polyakov} = \frac{f}{8} Re Pr \left[\frac{1 + 0.83e^2}{1 + 0.042e^2 [E^{1/4} \log_{10}(Re/8)]^{-1}} + 12.7 \sqrt{\frac{f}{8}} \left\{ Pr^{2/3} \left(1 + \frac{0.72e^3(1 + 0.28\sqrt{e})}{1 + 0.43e^4} \right) - \frac{1 + 0.58e^2}{1 + 0.83e^2} \right\} \right]^{-1}, \quad (9)$$

where

$$e = \frac{10^3 Gr_q}{Re^{2.75} Pr}, \quad f = \left[\frac{1 + 0.83e^2}{1.82 \log_{10}(Re/8) + 0.076e^2 E^{-0.25}} \right]^2$$

2.2.2. Jackson, Cotton and Axcell (1989)

In a 1989 review paper on mixed convection in a vertical channel, Jackson, Cotton and Axcell summarized the large body of research work performed in the area until then [17]. Eq. (10) gives the correlation that was suggested in their review. Experimental data with water, air, mercury and supercritical carbon dioxide were presented and the range of buoyancy parameters (Bo^*) extends from 10^{-7} to 10^{-2} . The correlation is discontinuous near $Bo^* \sim 3 \times 10^{-6}$ and the correlation form is implicit.

$$\frac{Nu_{Jackson}}{Nu_F} = \left(1 - \frac{8 \times 10^4 Bo^*}{(Nu_{Jackson}/Nu_F)^2} \right)^{0.46} \quad (10)$$

2.2.3. Vilemas, Poskas and Kaupas (1992)

Vilemas, Poskas and Kaupas developed an empirical correlation for air in the mixed convection regime [18]. The data range covered Re_{in} numbers from 3000 to 50,000, q_{in}^+ from 0.00035 to 0.0024 and a new buoyancy parameter (Eq. (11)) from 8×10^{-6} to 3.37×10^{-3} (inlet values). It should be noted that this correlation is discontinuous in the area of $K_{in2} < K_{in} < K_{in3}$ due to high sensitivity of the flow to small changes in the buoyancy parameter. Therefore, when we evaluate the Nusselt number in this region for comparison to the data we will use their correlation from $K_{in1} < K_{in} < K_{in2}$ as an approximate value.

$$K = \frac{Gr_q}{4Re^3 Pr}, \quad (11)$$

when $K_{in} < K_{in1} Nu_{Villemas} = Nu_F$,

$$\text{when } K_{in1} < K_{in} < K_{in2} Nu_{Villemas} = \frac{Nu_F}{0.98 + 0.54K_{in}^{0.25} (x/D)^{0.975}},$$

when $K_{in2} < K_{in} < K_{in3}$ not available,

when $K_{in3} < K_{in}$

if $0 < x/D < x_{min}/D$,

$$Nu_{Villemas} = Nu_F \left[11.3K_{in}^{1/3} + (5.39 \times 10^{-9} + 5.12 \times 10^{-4} \ln(K_{in})) \left(\frac{x}{D} - \frac{x_{min}}{D} \right)^2 \right],$$

if $x_{min}/D \leq x/D \leq x_{max}/D$,

$$Nu_{Villemas} = Nu_F \left[11.3K_{in}^{1/3} + (3.865 \times 10^{-9} + 3.672 \times 10^{-4} \ln(K_{in})) \left(\frac{x}{D} - \frac{x_{min}}{D} \right) \right],$$

if $x_{max}/D < x/D$, $Nu_{Villemas}$

$$= Nu_F \left[14.5K_{in}^{1/3} - 3.555K_{in}^{0.92} q_{in}^{+0.26} \left(\frac{x}{D} - \frac{x_{max}}{D} \right)^{0.46} \right],$$

where $K_{in1} = 2$ to 4×10^{-6} , $K_{in2} = 2.5 \times 10^{-5}$,

$$K_{in3} = 1.5 \times 10^{-3} q_{in}^{+0.45}, \quad x_{min}/D = 5.19 + 0.0059/K_{in}^{0.9},$$

$$x_{max}/D = 8.8 + 0.063/K_{in}^{0.7}. \tag{12}$$

2.2.4. Celeta et al. (1998) and Aicher and Martin (1996)

Celeta et al. [19] combined the results of Aicher and Martin’s [20] with those of Jackson and Hall [5] to produce a new correlation. When Aicher and Martin were developing their correlation, they introduced a new idea, which was not recognized in earlier studies. They correlated the upward heated flow Nusselt number with the downward heated flow Nusselt number to develop a smooth functional form. This idea was adopted by Celeta et al.; they modified the form of Aicher and Martin to fit their own data better and included an axial effect, which had been mentioned by Aicher and Martin but was not included in their correlation. Celeta et al. utilized the buoyancy parameter Bo^* , developed by Jackson and Hall, as their governing non-dimensional number. Eq. (13) is their correlation. Celeta et al. originally developed their correlation by using the Dittus–Boelter correlation with a correction factor for the water properties variation as a forced convection Nusselt number. Within this study, we will use the Gnielinski correlation [16] with the correction factor for gas properties variation as our Nu_F instead when we evaluate the Celeta et al. correlation. The limitation of the correlation is not explicitly described in the paper by Celeta et al. However, their experimental data cover Reynolds numbers from 800 to 23,000, buoyancy parameters $Bo^* < 0.156$ and $L/D < 60$.

$$Nu_{Celeta} = \sqrt{Nu_F^2 + Nu_N^2} \left\{ 1 - \left(0.36 + 0.0065 \frac{x}{D} \right) \times \exp \left(-0.8 \ln \frac{8 \times 10^4 Bo^*}{869(x/D)^{-2.16}} \right) \right\}, \tag{13}$$

where

$$Nu_F = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b} \right)^{-0.45},$$

$$Nu_N = \frac{0.15(Gr_{\Delta T,w} Pr_w)^{1/3}}{\left(1 + (0.437/Pr_w)^9 \right)^{16/27}}.$$

The subscript “w” indicates that properties are evaluated at the wall temperature.

3. Data comparison to earlier correlations

Since details of the experimental facility and the data can be found in [1–3,21], they are not repeated in this paper. Before presenting these comparisons, an indicator will be defined to measure the degree of agreement between a correlation and the experimental data. The “R-square” is a statistical quantity, which is defined as a ratio of the sum of squares of the regression (SSR) to the sum of squares about the mean (SST). The R-square is defined as the first definition in Eq. (14). The second definition in Eq. (14) can be derived from the first, when the sum of the errors due to the regression (SSR) is equal to the difference between SST and SSR. This condition holds only when the data are regressed via the least mean square method. However, since the correlations that we are testing can be approximated as a result of the regression of the data, the second definition is used in our study. The symbol Nu_{EXP} is defined as the mean value of the measured Nusselt number for a specified heat transfer regime and N is the total number of data in that regime,

$$R^2 = \frac{\sum_{i=1}^N (Nu_{Correlation,i} - \overline{Nu_{EXP}})^2}{\sum_{i=1}^N (Nu_{EXP,i} - \overline{Nu_{EXP}})^2} = 1 - \frac{\sum_{i=1}^N (Nu_{Correlation,i} - Nu_{EXP,i})^2}{\sum_{i=1}^N (Nu_{EXP,i} - \overline{Nu_{EXP}})^2}. \tag{14}$$

Only the best performing correlation will be shown on a comparison figure. These figures have as the X-axis the prediction of the correlation and as the Y-axis the measured Nusselt number. The more data that are concentrated near $Y=X$ line, the better is the performance of the correlation and the value of the two indicators becomes closer to unity.

First, we will only compare the mixed convection laminar data to the correlations (i.e., Eqs. (5)–(7)). Fig. 1 shows the performance of the best fitting correlation, “Worsøe-Schmidt”. All three correlations have negative values of R-square, which means the predictions of the mixed convection laminar correlations available in the literature are not satisfactory in comparison to our gas experimental data. This discrepancy is mostly because these correlations were based on water experiments and are not necessarily meant to fit the mixed convection laminar “gas” flow. However, it is surprising that the correlations of Worsøe–Schmidt do not agree with the data even though the numerical analyses were performed with gas properties and the data were within the numerical analyses range. This disagreement may be due to the relatively higher uncertainty (~25%) in the helium measurement compared to the data from other gases (below 10%) (nitrogen and carbon dioxide) [1].

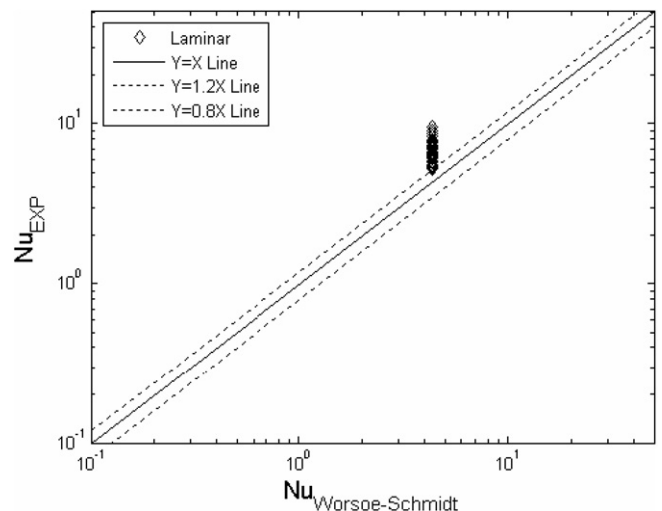


Fig. 1. Comparison of correlation by Worsøe-Schmidt to the helium data.

Table 1 summarizes the comparison to the data in terms of R-square for all the data in turbulent and DTHT heat transfer regimes separately. Examining the table, one sees that the best existing correlation in terms of R-square is the one by Celeta et al. Fig. 2 presents the comparison results for this correlation. Surprisingly, their correlation performs better in the acceleration driven DTHT regime than in the buoyancy driven DTHT regime (see Table 1). However, there is a problem with it, since the correlation was developed for a shorter test section ($L/D \sim 60$) than our experimental apparatus ($L/D \sim 115$), the predicted Nusselt number is much smaller (even smaller than the forced convection laminar Nusselt number of about 4.36) than the measured value in the buoyancy induced DTHT regime when x/D exceeds the range of their experiment. Another reason is possibly due to the characteristic of gas flow; “re-turbulizing” cases did not occur with water experiments, since gas and water property variations differ [21]. Therefore, their correlation can underpredict the heat transfer coefficient in certain situations when it is applied to gases.

Since gases have different directions for property variations with temperature compared to liquids or supercritical fluids, as was demonstrated in [21], most of the DTHT correlations that were developed with liquids or supercritical fluids tend to overpredict the Nusselt number, except for that of Celeta et al. However, Celeta et al. underpredict the heat transfer coefficient for “re-turbulizing flow”. In addition, since their correlation involves properties based on the wall temperature when estimating the free convection Nusselt number (see Eq. (13)), it is not easy to apply for estimating the wall temperature. Therefore, a new correlation that can successfully fit the gas DTHT regime with a reasonable agreement to the laminar, transition, turbulent and DTHT heat transfer data is desirable. In addition, the requirement for wall temperature information should be minimized to simplify the calculation.

4. Development of new correlations

The features desired for a new correlation are as follows: (1) It should reflect the physical phenomena. (2) Most of the physical properties should be evaluated at the bulk temperature. (3) It should have an explicit form to minimize iterations. (4) It should cover all heat transfer regimes including the laminar, turbulent and DTHT as well as mixed convection laminar.

In the non-dimensional Navier–Stokes equations (refer to Eq. (2)), the parameter which represents the buoyancy effect is Gr_q/Re^2 (also called the Richardson number) for cases with specified wall heat flux. Fig. 3 shows the correlation between this parameter and the ratio of Nu_{EXP} to the Nusselt number predicted by Eq. (15) for forced convection laminar heat transfer correlation shown in Eq. (15) (Refs. [1 and 7] discuss this prediction further). It can be observed from Fig. 3 that the ratio of the measured to predicted Nusselt number correlation agrees well with the Gr_q/Re^2 parameter. Therefore, Gr_q/Re^2 was selected for correlating the non-dimensional parameter for buoyancy effect in laminar mixed-convection,

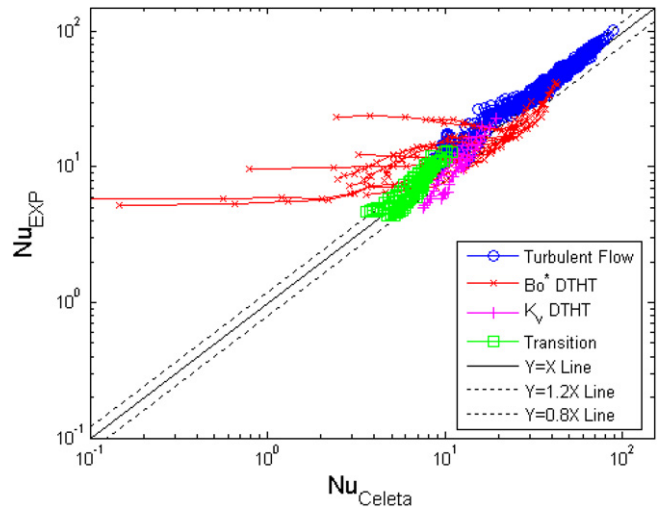


Fig. 2. Comparison of correlation by Celeta et al. to the turbulent data.

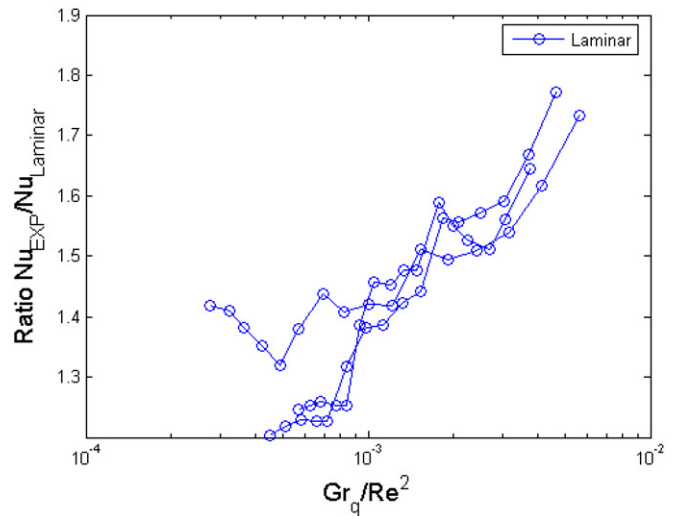


Fig. 3. Performance relative to Gr_q^2/Re for laminar mixed convection.

$$Nu_{Laminar} = \left(\frac{1}{Nu_{\infty}} - \frac{1}{2} \sum_{m=1}^{10} \frac{\exp(-\gamma_m^2 x^+)}{A_m \gamma_m^4} \right)^{-1},$$

where $Nu_{\infty} = 4.364$, $x^+ = \frac{2x/D}{RePr}$, $\gamma_m = 4m + \frac{4}{3}$,
 $A_m = 0.4165 \gamma_m^{-7/3}$. (15)

Regarding correlation development for the DTHT regimes, the DTHT correlation should diverge from the forced convection correlation when certain criteria are met, as one can observe from the trends

Table 1
R-square values for various existing and new correlations

Correlation/Regime	All	Turbulent	Bo+ DTHT	Kv DTHT	Transition	Laminar
Petukhov (Eq. (9))	0.8873	0.9489	−1.3321	−2.0029	−3.3808	N/A
Jackson (Eq. (10))	0.9330	0.9326	0.0858	0.7098	0.6711	N/A
Vilemas (Eq. (12))	0.4917	0.2009	−0.4386	−0.4416	−0.0787	N/A
Celeta (Eq. (13))	0.9405	0.9323	0.3425	0.7097	0.7059	N/A
New (Type-1)	0.9602	0.9338	0.8811	0.8710	0.8096	0.8911
New (Type-2)	0.9763	0.9637	0.8813	0.9032	0.6864	0.8911
New (Type-3)	0.9512	0.9203	0.8151	0.9320	0.7873	0.8911

of experimental data (see also [1–3] for more information). A functional form for the DTHT regimes with such characteristics can be obtained by revising the constant in the Gnielinski correlation, as demonstrated in Eq. (16). This technique follows the Gnielinski approach [16] to correlate the data better, near the laminar to turbulent transition,

$$Nu_{Gnielinsky} = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b}\right)^{-0.45} \left(1 + \frac{x}{D}\right)$$

$$\rightarrow Nu_{NEW} = \frac{(f/8)(Re - F(X))Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b}\right)^{-0.45} \left(1 + \frac{x}{D}\right). \quad (16)$$

If $F(X)$ differs from 1000 and can be related to a non-dimensional number (X) which captures the key physical phenomena for a particular case, then one can expect that the predicted Nusselt number will behave as the experimental data do. Since the DTHT in the current data is due to the buoyancy and/or acceleration effects, relevant non-dimensional parameters for $F(X)$ are selected to be the buoyancy and acceleration parameters.

Another desirable feature for the new correlation is that the condition for determining each regime (laminar, turbulent, DTHT and so forth) is based on logical inlet non-dimensional numbers, such as the Reynolds number, buoyancy and acceleration parameters. A comparable approach for determination of regime change can be found in the work of Vilemas et al. (see Eq. (12)). This treatment is useful because the Reynolds number and the buoyancy and acceleration parameters all have maximum values at the inlet of the heated section.¹ Therefore, in this case the heat transfer regime is mainly governed by the inlet values of the dimensionless numbers and the correlation may be chosen based on the same criteria. This scheme is also consistent with the heat transfer regime map developed in [1–3]. The inlet thresholds for re-turbulization and the buoyancy driven and acceleration driven DTHT are already discussed in [1–3].

However, since re-turbulization occurs in the middle of the channel [1–3] and the slope of $Nu(Re)$ with distance changes after the re-turbulization point, a second re-turbulization threshold has to be deduced in terms of the local buoyancy parameter. This is because re-turbulization occurs when turbulent heat transport near the wall increases downstream in a heated channel where the buoyancy force decreases below a certain limit. It was observed that most cases begin to recover to normal turbulent heat transfer after the local buoyancy parameter decreases below $Bo^* \sim 6 \times 10^{-7}$. This condition will be used for developing the correlation and further dividing the heat transfer regimes.

The basic procedure for developing this correlation starts by correlating $F(X)$ to an appropriate non-dimensional number for each regime with a test function as seen in Eq. (17). Two functional forms are tested to minimize the number of coefficients to determine the fit to experimental data. The coefficients C_1 and C_2 are to be obtained by matching the data and X is the governing non-dimensional number for each regime

$$F(X) = C_1(X)^{C_2} \quad \text{and/or} \quad F(X) = C_1 \log_{10}(C_2 X). \quad (17)$$

Since the governing non-dimensional numbers (the buoyancy and acceleration parameters) vary by orders of magnitude while the function response range is much smaller, the exponential power law and logarithmic form were chosen. These two functional forms provide an advantage over other forms, such as polynomial forms, as they can accommodate large variation of a variable while yielding a small functional response.

After an extensive search and fitting procedure to identify appropriate values C_1 and C_2 that would correlate best with the

data for each heat transfer regime, a set of correlations that covers forced convection laminar, mixed-convection laminar, turbulent, acceleration driven DTHT, re-turbulizing buoyancy driven DTHT and non-returbulizing buoyancy driven DTHT regimes was developed. In these correlations all non-dimensional numbers are evaluated at the bulk temperature. It is noted that forced laminar Nusselt number ($Nu_{Laminar}$) is calculated from Eq. (15) and the mixed convection laminar regime remains the same for all three sets of correlations.

Type-1 correlation set

If $Re_{in} \leq 2300$ (Mixed convection laminar and forced convection laminar)

$$Nu_{TYPE1-Laminar} = \max\left(1, 3.0\left(\frac{Gr_q}{Re^2}\right)^{0.11}\right) Nu_{Laminar}.$$

If $K_{v_{in}} < 2.5 \times 10^{-6}$, $Bo_{in}^* < 2.0 \times 10^{-6}$ and $Re_{in} > 2300$ (Turbulent)

$$Nu_{TYPE1} = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b}\right)^{-0.45} \left(1 + \frac{x}{D}\right)$$

$$= Nu_{Gnielinsky}.$$

If $K_{v_{in}} \geq 2.5 \times 10^{-6}$, $Bo_{in}^* < 2.0 \times 10^{-6}$ and $Re_{in} > 2300$ (K_v DTHT)

$$Nu_{TYPE1-temp} = \frac{(f/8)(Re - 0.185K_v^{-2/3})Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b}\right)^{-0.45} \left(1 + \frac{x}{D}\right).$$

If $K_{v_{in}} < 2.5 \times 10^{-6}$, $3.5 \times 10^{-6} > Bo_{in}^* \geq 2.0 \times 10^{-6}$ and $Re_{in} > 2300$

If $Bo^* \geq 6.0 \times 10^{-7}$ (Re-turbulizing Bo^* DTHT before re-turbulization point)

$$Nu_{TYPE1-temp} = \frac{(f/8)(Re - 1.45 \times 10^{-7} Bo^{*-1.7})Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b}\right)^{-0.45} \left(1 + \frac{x}{D}\right).$$

If $Bo^* < 6.0 \times 10^{-7}$ (Re-turbulizing Bo^* DTHT after re-turbulization point)

$$Nu_{TYPE1-temp} = \frac{(f/8)(Re - 8.34 \times 10^7 Bo^{*0.69})Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b}\right)^{-0.45} \left(1 + \frac{x}{D}\right).$$

If $K_{v_{in}} < 2.5 \times 10^{-6}$, $Bo_{in}^* \geq 3.5 \times 10^{-6}$ and $Re_{in} > 2300$ (Non-returbulizing Bo^* DTHT)

$$Nu_{TYPE1-temp} = \frac{(f/8)(Re - 79.4Bo^{*-0.28})Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b}\right)^{-0.45} \left(1 + \frac{x}{D}\right),$$

$$Nu_{TYPE1} = \max(Nu_{TYPE1-temp}, Nu_{Laminar}),$$

where $f = (1.82 \log_{10} Re - 1.64)^{-2}$ when $Re_{in} \geq 2300$.

Performance of the Type-1 correlation set is shown in Fig. 4.

For the DTHT and laminar convection regimes, a few characteristics of the correlation should be discussed. First, since the deteriorated turbulent heat transfer coefficient cannot be lower than the forced laminar convection, the DTHT regime heat transfer coefficient is the maximum between the modified Gnielinski correlation and the forced laminar convection correlation. This requirement provides a lower bound for the correlation. A heat transfer correlation for the transition regime is not developed separately since we were not able to identify governing parameters in the laminar to turbulent flow transition successfully.

For laminar mixed-convection, a multiplier to the forced convection laminar correlation was developed with the governing non-dimensional number, Gr_q/Re^2 . The power and the leading coefficients are also determined from empirical curve fitting. The multiplication factor is the maximum value between unity and the function developed to provide a lower bound to the correlation. Therefore, the correlation can cover both the forced convection laminar and the mixed-convection laminar cases.

¹ This holds for gas flow only, since the viscosity of gas increases with temperature while that of water decreases, leading to increased Reynolds number along the heated section in the case of water [9].

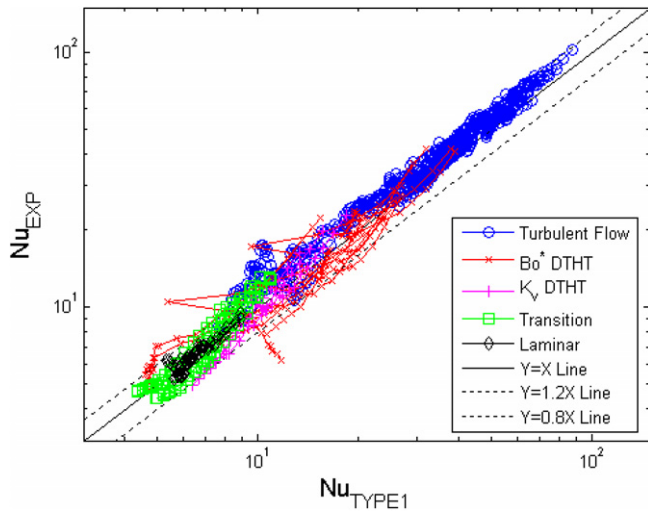


Fig. 4. Performance of Type-1 correlation set.

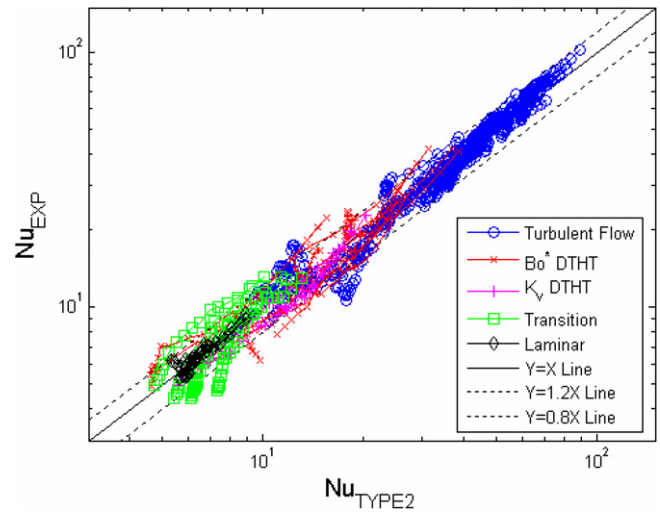


Fig. 5. Performance of Type-2 correlation set.

The term $(T_w/T_b)^{-0.45}$ in the original Gnielinski form makes prediction of the heat transfer coefficient and wall temperature difficult, since it requires iteration on an unknown wall temperature. To avoid the iteration, another set of correlations with the term $(T_w/T_b)^{-0.5}$, which allows one to solve for wall temperature analytically, was developed. The friction factor that is used for the Gnielinski correlation is valid only for $Re > 10,000$ (Filonenko correlation). Therefore, the friction factor needs to be reconsidered when the Gnielinski correlation is applied to $Re < 10,000$. The turbulent Blasius friction factor is commonly used for lower Reynolds numbers and is valid for $Re > 4000$. For a Reynolds number between 4000 and 2300 the friction factor will be estimated by interpolation between the Blasius friction factor at 4000 and the laminar friction factor at 2300. The following set of correlations is called the Type-2 correlation. The power law of the governing non-dimensional numbers fitted better in the Type-1 correlation while the logarithmic function of the governing non-dimensional number performed better in the Type-2 concept. Again the transition regime was not separated and the laminar correlation is the same, since it does not include a temperature ratio multiplier. Even though the iteration for estimating wall temperature is eliminated, dividing into many heat transfer regimes and having a separate correlation for each regime is still complicated for engineering applications.

Type-2 correlation set

If $K_{v,in} < 2.5 \times 10^{-6}$, $Bo_{in}^* < 2.0 \times 10^{-6}$ and $Re_{in} > 2300$ (Turbulent),

$$Nu_{TYPE2} = \frac{(f/8)(Re + 500)Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b}\right)^{-0.5} \left(1 + \frac{x}{D}\right).$$

If $K_{v,in} \geq 2.5 \times 10^{-6}$, $Bo_{in}^* < 2.0 \times 10^{-6}$ and $Re_{in} > 2300$ (K_v DTHT),

$$Nu_{TYPE2-temp} = \frac{(f/8)[Re - 3500\log_{10}(3.8 \times 10^5 K_v)]Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \times \left(\frac{T_w}{T_b}\right)^{-0.5} \left(1 + \frac{x}{D}\right).$$

If $K_{v,in} < 2.5 \times 10^{-6}$, $3.5 \times 10^{-6} > Bo_{in}^* \geq 2.0 \times 10^{-6}$ and $Re_{in} > 2300$,
If $Bo^* \geq 6.0 \times 10^{-7}$ (Re-turbulizing Bo^* DTHT)

$$Nu_{TYPE2-temp} = \frac{(f/8)[Re + 6500\log_{10}(4.3 \times 10^5 Bo^*)]Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \times \left(\frac{T_w}{T_b}\right)^{-0.5} \left(1 + \frac{x}{D}\right).$$

If $Bo^* < 6.0 \times 10^{-7}$ (Re-turbulizing Bo^* DTHT),

$$Nu_{TYPE2-temp} = \frac{(f/8)[Re - 3900\log_{10}(2 \times 10^7 Bo^*)]Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \times \left(\frac{T_w}{T_b}\right)^{-0.5} \left(1 + \frac{x}{D}\right).$$

If $K_{v,in} < 2.5 \times 10^{-6}$, $Bo_{in}^* \geq 3.5 \times 10^{-6}$ and $Re_{in} > 2300$ (Bo^* DTHT),

$$Nu_{TYPE2-temp} = \frac{(f/8)[Re + 2000\log_{10}(1.6 \times 10^4 Bo^*)]Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \times \left(\frac{T_w}{T_b}\right)^{-0.5} \left(1 + \frac{x}{D}\right),$$

$$Nu_{TYPE2} = \max(Nu_{TYPE2-temp}, Nu_{Laminar}).$$

If $Re \geq 10,000$ $f = (1.82 \log_{10} Re - 1.64)^{-2}$ (Filonenko),

If $10,000 > Re \geq 4000$ $f = 0.314/Re^{0.25}$ (Blasius),

If $4000 > Re \geq 2300$ $f = 0.012 + 6.86 \times 10^{-6} Re$,

If $2300 > Re$ $f = 64/Re$.

Performance of the Type-2 correlation set is shown in Fig. 5.

The Type-3 correlation, discussed below, utilizes a new non-dimensional number that combines the acceleration and the buoyancy effects. The basic idea is finding the best ratio between the non-dimensional heat flux (q^+) and the Reynolds number, since both the acceleration and buoyancy effects can be expressed as a combination of these two dimensionless numbers [1]. The new heat transfer regime map, presented in [1–3], uses q^+ and the Reynolds number as the main non-dimensional parameters and successfully separates each regime. Therefore, finding one non-dimensional number, which is a combination of q^+ and the Reynolds number with an appropriate relationship, to capture both acceleration and buoyancy effects is a reasonable approach to further simplify the correlation.

The resulting non-dimensional number is $q^+/Re^{0.44}$, which was found to provide the best ratio for correlating the data. This parameter shifts the buoyancy induced DTHT and the acceleration induced DTHT data in such a way that they overlap each other. Thus, the new number makes it possible to correlate both effects

² This relation provides linear interpolation between the laminar friction factor at $Re = 2300$ and the Blasius friction factor at $Re = 4000$.

with a single number and to derive a simplified correlation. We will call this new non-dimensional number a “DTHT” number in order to distinguish it from the property dimensionless group, which was developed in [1–3] when the buoyancy parameter was being re-arranged as a combination of the non-dimensional heat flux, Reynolds number and property group.

Another advantage of the Type-3 correlation is that interference between the acceleration effect and the buoyancy effect is automatically addressed by using only one non-dimensional number. The flow regime that overlaps between the buoyancy induced DTHT and the acceleration induced DTHT was not covered in our experiments, thus the regime is still not well defined. Since most literature deals with only one effect at a time, the DTHT regime with overlapping acceleration and buoyancy effects is a new regime that requires further attention. In the meantime, the DTHT number avoids this problem to some extent, as both effects are included in one non-dimensional number. The following set of equations depicts the Type-3 correlation. The friction factor was calculated with Filonenko’s correlation for simplicity, while the non-iterative form of Gnielinski correlation with $(T_w/T_b)^{-0.5}$ is used for the normal turbulent heat transfer.

Type-3 correlation set

If $K_{vin} < 2.5 \times 10^{-6}$, $Bo_{in}^* < 2.0 \times 10^{-6}$ and $Re_{in} > 2300$ (Turbulent)

$$Nu_{TYPE3} = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b}\right)^{-0.5} \left(1 + \frac{x}{D}\right)^{-2/3}$$

If $K_{vin} \geq 2.5 \times 10^{-6}$ or $Bo_{in}^* \geq 2.0 \times 10^{-6}$ and $Re_{in} > 2300$ (DTHT)

$$Nu_{TYPE3-temp} = \frac{(f/8) \left(Re - 0.011 \left(\frac{q^+}{Re^{0.44}}\right)^{-1.16}\right) Pr}{1 + 12.7\sqrt{f/8}(Pr^{2/3} - 1)} \left(\frac{T_w}{T_b}\right)^{-0.5} \left(1 + \frac{x}{D}\right)^{-2/3}$$

$$Nu_{TYPE3} = \max(Nu_{TYPE3-temp}, Nu_{Laminar})$$

Performance of the Type-3 correlation set is shown in Fig. 6.

Table 1 also shows the performance of the new correlations. Compared to the other four mixed convection laminar relations, the laminar correlation agrees with the experimental data more successfully. However, as mentioned before, more experimental data for the mixed convection laminar regime will be required to increase confidence in the correlation developed in this paper. Future experiments should be performed with test sections of appro-

appropriate diameter and gases other than helium, such as nitrogen or carbon dioxide at low flow rates, to operate in the laminar flow regime.

Overall, the Type-1 and Type-2 correlations (Figs. 4 and 5) perform best over all heat transfer regimes and Type-3 (Fig. 6) shows satisfactory performance as well. Comparing the three new correlations against the best performing existing correlation, e.g., the correlation of Celeta et al., it is clear that all new correlation sets perform better in all regimes. Therefore, it is concluded that all three correlation sets can predict gas heat transfer coefficients better than the existing correlations.

A limitation in the use of these correlations is to stay within the range covered in the experiments, since the correlations were developed from the data presented in [1–3] only. It would be desirable to broaden the range of data by using more existing archival data. However, the gas heat transfer experiments presented in the literature are limited. Furthermore, much of the available data in these regimes were taken with water and supercritical fluids, which should not be included since water and supercritical fluids have different property variations than gases [21], resulting in different streamwise variation of the governing parameters.

Other restrictions for applying the correlation sets developed are: (1) The interference between the acceleration effect and the buoyancy effect is not fully covered for the cases where the operating conditions are above both the acceleration threshold and the buoyancy threshold at the same time. The relative influences of the buoyancy effect and the acceleration effect on turbulent flow are not fully treated yet. More experimental data and theoretical development should be followed to further understand this complex regime. Some theories, such as that presented by Petukhov and Polyakov [4], cover both effects at the same time but the authors implied that their theory can explain this regime only within a limited range. (2) Since there is a potential for having different governing physical phenomena depending on geometrical conditions, great caution should be exercised when using the correlation set in applications that exhibit significant differences in geometry of the heated section from this experimental facility. The gas heat transfer data in the DTHT regimes showed that fully established, turbulent heat transfer was rarely observed under our experimental conditions even though our test section has a relatively large heated length (up to $L/D \sim 115$). Since the quasi-developed region, where the local parameters alone can successfully describe the turbulent behavior, is rarely observed in the DTHT cases, the sensitivity of the turbulent flow structure to the geometry of the channel and the upstream conditions is stronger for the DTHT flow than the normal fully developed turbulent flow.

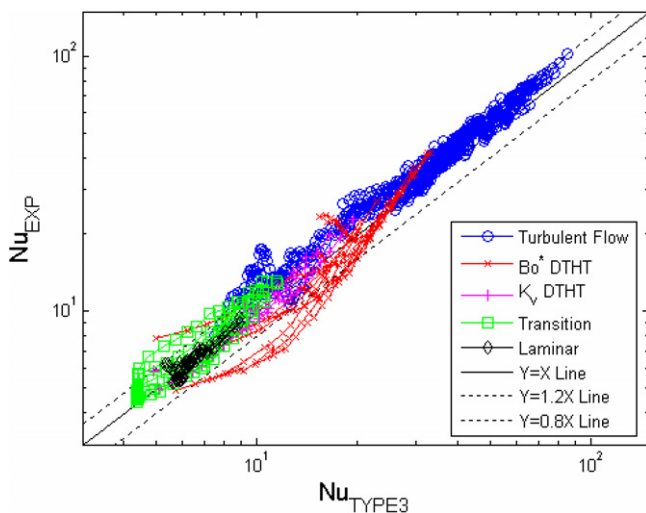


Fig. 6. Performance of Type-3 correlation set.

5. Summary

The recent data presented in [1–3], which covered the mixed convection laminar, turbulent, buoyancy driven DTHT and acceleration driven DTHT regimes, were compared to existing correlations. The mixed convection laminar correlations selected did not agree well with our data in this regime. However, the non-dimensional number $(=Gr_q/Re^2)$, selected in this paper as a key parameter for the mixed-convection laminar heat transfer regime, led to a reasonable correlation of the helium data in [1–3].

For the forced turbulent and the DTHT regimes, the correlation by Celeta et al. performed the best among the existing correlations. However, it sometimes underpredicts the heat transfer coefficient due to the limited L/D range of the database and the differences in the characteristics of operating fluids (water vs. gas). Therefore, to design gas heat transfer systems that operate in these regimes with confidence, it was necessary to develop a new correlation that can cover the turbulent and DTHT regimes better.

The approach adopted in this paper is to use the Gnielinski correlation, which is widely recommended for turbulent convection, and to modify its constants by an empirically fitted function that accounts for the physical phenomena that drive the DTHT regimes. Thus, the function depends on the buoyancy parameter or the acceleration parameter, in accordance with the operating heat transfer regime. Three sets of correlation sets have been developed. Type-1 and Type-2 sets correlate the data more accurately but are rather complex, since they are divided into many sub-regimes. The Type-3 correlation set has the simplest form, which was made possible through the development of a new non-dimensional number, designated as the “DTHT” number. This “DTHT” number successfully captures both the buoyancy and acceleration effects and simplifies the correlation form while maintaining reasonable accuracy as shown in Table 1 and Fig. 6. All of the new correlation sets are valid over our range of experimental data: (1) inlet Reynolds number above 1800 (2) inlet Bo^* to 1×10^{-5} and (3) inlet K_v to 5×10^{-6} .

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References

- [1] J.I. Lee, P. Hejzlar, P. Saha, P. Stahle, M.S. Kazimi, D.M. McEligot, Deteriorated turbulent heat transfer of gas up-flow in a circular tube: experimental data, *Int. J. Heat Mass* 51 (2008) 3259–3266.
- [2] J.I. Lee, P. Hejzlar, P. Stahle, P. Saha, D.M. McEligot, G.E. McCreery, R.R. Schultz, Design of thermal-hydraulic loop related to advanced gas-cooled reactor, Third Annual Report, MIT-GFR-042, INL/EXT-06-11801, Idaho National Laboratory and Massachusetts Institute of Technology, Department of Nuclear Science and Engineering, September 2006.
- [3] J.I. Lee, P. Hejzlar, M.S. Kazimi, Deteriorated turbulent heat transfer to up-flowing gas in a heated vertical channel, MIT-ANP-TR-115, Center for Advanced Nuclear Energy Systems, Massachusetts Institute of Technology, May 2007.
- [4] B.S. Petukhov, A.F. Polyakov, *Heat Transfer in Turbulent Mixed Convection*, Hemisphere, Washington, DC, 1988.
- [5] W.B. Hall, J.D. Jackson, Laminarization of a Turbulent Pipe Flow by Buoyancy Forces, *ASME 69-HT-55* (1969).
- [6] A.S. Monin, A.M. Yaglom, *Statistical Fluid Mechanics*, vol. 1, MIT Press, Cambridge, MA, 1979.
- [7] W.M. Kays, M.E. Crawford, *Convective Heat and Mass Transfer*, third ed., McGraw-Hill, New York, 1993.
- [8] D.M. McEligot, C.W. Coon, H.C. Perkins, Relaminarization in tubes, *Int. J. Heat Mass Transfer* 13 (1969) 431–433 (Short Communication).
- [9] D.M. McEligot, J.D. Jackson, Deterioration criteria for convective heat transfer in gas flow through non-circular ducts, *Nucl. Eng. Des.* 232 (2004) 327–333 (Short Communication).
- [10] C.A. Bankston, The transition from turbulent to laminar gas flow in a heated pipe, *J. Heat Transfer* (1970) 569–579.
- [11] T.M. Hallman, Combined forced and free-laminar heat transfer in vertical tubes with uniform internal heat generation, *Trans. Am. Soc. Mech. Engrs.* 78 (1955) 1831–1841.
- [12] T.M. Hallman, Experimental study of combined forced and free laminar convection in a vertical tube, NASA Technical Note, TN D-1104, December 1961.
- [13] P.M. Worsøe-Schmidt, G. Leppert, Heat transfer and friction for laminar flow of gas in a circular tube at high heating rate, *Int. J. Heat Mass Transfer* 8 (1965) 1281–1301.
- [14] P.M. Worsøe-Schmidt, Heat transfer and friction for laminar flow of helium and carbon dioxide in a circular tube at high heating rate, *Int. J. Heat Mass Transfer* 9 (1966) 1291–1295.
- [15] S.W. Churchill, Combined free and forced convection in channels, *Heat Exchanger Design Handbook*, Begell House, 1998 (Chapter 2.5.10).
- [16] V. Gnielinski, New equations for heat and mass transfer in turbulent pipe and channel flow, *Int. Chem. Eng.* 16 (2) (1976) 359–387.
- [17] J.D. Jackson, M.A. Cotton, B.P. Axcell, Studies of mixed convection in vertical tubes, *Int. J. Heat Fluid Flow* 10 (1989) 2–15.
- [18] J.V. Vilemas, P.S. Poškas, V.E. Kaupas, Local heat transfer in a vertical gas-cooled tube with turbulent mixed convection and different heat fluxes, *Int. J. Heat Mass Transfer* 35 (10) (1992) 2421–2428.
- [19] G.P. Celata, F. D’Annibale, A. Chiaradia, M. Cumo, Upflow turbulent mixed convection heat transfer in vertical pipes, *Int. J. Heat Mass Transfer* 41 (1998) 4037–4054.
- [20] T. Aicher, H. Martin, New correlation for mixed turbulent natural and forced convection heat transfer in vertical tubes, *Int. J. Heat Mass Transfer* 40 (15) (1997) 3617–3626.
- [21] J.I. Lee, P. Hejzlar, P. Saha, M.S. Kazimi, Studies of the deteriorated turbulent heat transfer regime for the gas-cooled fast reactor decay heat removal system, *Nucl. Eng. Des.* 237 (2007) 1033–1045.